An Adaptive Frame-Based Interpolation Method of Channel Estimation for Space-Time Block Codes in Moderate Fading Channels

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SUMMARY  The application of Orthogonal Space-Time Block Codes (O-STBC) as the encoding scheme in the presence of “non-quasi-static” fading was considered. A simple and efficient adaptive method of channel estimation based on the interpolation of estimates acquired at the pre-amble and post-amble of framed blocks of information is developed. Moreover, the proposed method is proven, both theoretically and by simulations, to outperform the alternative of channel tracking, despite its significant low complexity.

key words: O-STBC, channel estimation, interpolation, non-quasi-static fading, level-crossing ratio

1. Introduction

Since Space-Time Block Codes from Orthogonal Designs were introduced [2], as an extension of Alamouti Codes [3], they have gained a lot of attention due to their potential application in conjunction with multiple-input multiple-output (MIMO) systems.

Despite drawbacks in capacity, as shown in [4], O-STBC is very appealing in terms of complexity and performance. In addition to this, to the best of our knowledge, it is the only STBC technique that can be easily made robust to the continuous (rather than block) fluctuation characteristic of non-quasi-static fading channels [1], one of the causes of loss of orthogonality in symbol decoding.

Nevertheless, the performance of this improved scheme still relies on accurate channel state information (CSI), which may require the insertion of a large amount of pilot symbols, therefore degrading the overall throughput of the system.

Recently, channel estimation techniques for OFDM systems with multi-transmit diversity have been widely researched [6]–[11]. The approach given by [6] explores the STBC orthogonality to estimate the OFDM channel, though is shown to be efficient only in very low Doppler frequency scenarios. A Kalman filter approach to channel estimation of time-selective channels, given by [7]–[9], significantly improved the performance when compared to the non-tracking case. However, their performances in high signal-to-noise ratio (SNR) still lay far behind when compared to the case of perfect channel state information (CSI) present at the decoder. This is due to the small, though present, imperfections of the channel estimations.

The technique described in [10] has as drawback being computationally expensive due to the needed matrix inversion for every OFDM data symbols. Although being in a simpler form, the technique presented in [11], still has a relatively high complexity.

In this paper a simple and accurate method for channel estimation of Space-Time Block Coded OFDM systems, based on adaptive length frame-based interpolation method is proposed. The paper is structured as follows. In Sect. 2, the interpolation technique is justified by an analysis of the influence that the autocorrelation properties of the channel estimates have on the performance of the robust O-STBC scheme [1]. In Sect. 3, the adaptive frame-based interpolation scheme for channel estimation itself is explained. In Sect. 4, theoretical expressions of the BER for coherent schemes of the proposed and conventional systems using MPSK modulation are also derived. In Sect. 5, the performance of systems employing the proposed technique are compared to those of systems with perfect channel estimation and online channel tracking based on Kalman filter. Finally, conclusions are drawn in Sect. 6.

2. Interference Coefficient Analysis

In this section we analyze the influence of the autocorrelation of the channel estimation on minimizing the interference coefficient resultant from channel estimation errors within a symbol block. In addition to it, we demonstrate that the Linear Maximum Likelihood Decoder [1] benefits from a high-correlated channel estimation, therefore favoring the use of interpolation methods.

As described by [1], fully orthogonal linear symbol decoding in the presence of “non-quasi-static” fading channels (hereafter referred as moderate fading) is achieved by successively applying a partial orthogonal decoder, also derived in [1]. Here, moderate fading should be understood as a channel whose value is constant during one symbol transmission interval, but slightly changes between adjacent symbols. If STBC codewords are sent from four transmit
antennas in a moderate fading, the estimated symbol, say \( \hat{s}_1 \), after one step of the partially orthogonal combiner is given by

\[
\tilde{s}_{1,2} = s_1 h_{1,1} \tilde{h}_{1,6}^* + (s_2 h_{2,1} + s_3 h_{3,1} + s_4 h_{4,1} + w_1) \tilde{h}_{1,6}^* + (s_1 h_{1,2} + s_3 h_{3,2} + s_4 h_{4,2} + w_2) \tilde{h}_{2,5}^* + (s_1 h_{1,3} + s_4 h_{4,3} + s_2 h_{2,3} + w_3) \tilde{h}_{3,8}^* + (s_1 h_{1,4} + s_2 h_{2,4} + s_3 h_{3,4} + w_4) \tilde{h}_{4,7}^* + (s_1 h_{1,5} + s_2 h_{2,5} + s_3 h_{3,5} + s_4 h_{4,5} + w_5) \tilde{h}_{1,2}^* + (s_1 h_{1,6} + s_3 h_{3,6} + s_4 h_{4,6} + w_6) \tilde{h}_{2,1}^* + (s_1 h_{1,7} + s_4 h_{4,7} + s_2 h_{2,7} + s_3 h_{3,7} + w_7) \tilde{h}_{3,4}^* + (s_1 h_{1,8} + s_2 h_{2,8} + s_3 h_{3,8} + w_8) \tilde{h}_{4,3}^* (1)
\]

where \( \tilde{s}_{p,2} \) represents the estimation of \( s_p \), orthogonal to \( s_q \), \( h_{m,n} \) and \( \tilde{h}_{m,n} \) are the channel \( m \) at the transmission instant \( n \) and its estimation, respectively, and \( w \) denotes a zero-mean white complex Gaussian distributed noise with variance \( \sigma_n^2 / 2 \) per dimension.

By the analysis of Eq. (1) we can notice the presence of eight terms however the configuration adopted is a four transmit - one receive antenna MISO system. The reason for this lies in the fact that in one STBC block the set of symbols being transmitted is followed by its conjugated version, resulting in a rate \( 1 / 2 \) code when the number of transmit antennas is bigger than two. Also, it is clear that for perfect CSI at the decoder (\( h = \tilde{h} \)), the coefficient of \( s_2 \) vanishes, therefore leading to a partially orthogonal decoding \( \hat{s}_{1,2} \). Subsequently applying the partially orthogonal combiner, coefficients of \( s_3 \) and \( s_4 \) also vanish yielding a fully orthogonal decoding \( \hat{s}_{1,2,3,4} \). Unfortunately, decoders in the real world cannot count with perfect channel knowledge, thus the interference factor of \( s_q \) in the estimation of \( s_p \) does exist and depends on the accuracy of the estimation method.

Now, knowing that channel estimation errors are also responsible for loss of orthogonality in symbol decoding, one question could arise: what kind of estimation error would have less impact on the system performance? The one resulting from low-autocorrelated channel estimation methods, such as online tracking algorithms, or the one resulting from high-autocorrelated methods, such as polynomial interpolations?

To answer this question, we start by:

A. Modeling the estimated channel as

\[
\tilde{h}_{m,n} = (\sqrt{1 - \epsilon}) h_{m,n} + g_{m,n} (2)
\]

where \( g \) is the estimation error with mean square error (MSE) \( \epsilon \) normalized by the variance (power) of the \( h \) \( (\sigma_h^2) \). Details of (2) are found at the Appendix A.

B. Modeling each channel and estimation error processes as an auto-regressive model [7], therefore consecutive samples of the same process can be related by

\[
h_{m,n+u} = \alpha_{m,u} h_{m,n} + v_{m,n} (3)
\]

\[
g_{m,n+u} = \beta_{m,u} g_{m,n} + k_{m,n} (4)
\]

where \( v_n \) and \( k_m \) are zero-mean complex Gaussian random variables. Also, \( \alpha \) and \( \beta \) are the normalized autocorrelation functions of \( h \) and \( g \), respectively, and \( u \) is the instant difference between the left hand side and the right hand side of (3) and (4). Details of (3) and (4) are found in Appendix B.

C. Writing (1) in a simplified form

\[
\tilde{s}_{1,2} = s_1 + \frac{b s_2 + c s_3 + d s_4 + w}{a} (5)
\]

where \( a, b, c \) and \( d \) are factors which vary according to the values of \( h_{m,n} \) and \( w \) represents the sum of the noises in a symbol block.

Due to the multi-step nature of the decoder in [1] a full mathematical analysis of the interference coefficient present in the estimation of \( s_1 \) is prohibitively complex. Thus, we focus on the factors \( E[b] \) and \( E[a] \) after the first step of the partially orthogonal combiner, leading to a simpler though efficacious analysis. As \( b \) is existent due to channel estimation errors, it propagates through all the steps of the decoder, resulting in a final non-orthogonal estimation of \( s_1 \). Above, \( E[-] \) denotes expected value.

C.1. Gain Factor \( a \)

\[
a = (\beta_{1,4} g_{1,2}^* + z_1) h_{1,1} + g_{2,1} (a_2 h_{2,2}^* + r_2) + g_{1,2} (a_1 h_{1,1}^* + r_1) + (\beta_{2,4} g_{2,1}^* + z_2) h_{2,2} + (\beta_{3,4} g_{3,4}^* + z_3) h_{3,3} + g_{4,3} (a_4 h_{4,4}^* + r_4) + (\beta_{4,4} g_{4,4}^* + z_4) h_{4,4} + g_{4,3} (a_3 h_{3,3}^* + r_3) + (\sqrt{1 - \epsilon}) (h_{1,1} h_{1,6}^* + h_{2,2} h_{2,5}^* + h_{3,3} h_{3,8}^* + h_{4,4} h_{4,7}^* + h_{1,1} h_{1,2} + h_{2,2} h_{2,1} + h_{3,7} h_{3,4} + h_{4,8} h_{4,3}) (6)
\]

C.2. Interference Factor \( b \)

\[
b = (\beta_{1,4} g_{1,2}^* + z_2) h_{2,1} - g_{2,1} (a_1 h_{1,1}^* + r_3) + g_{1,2} (a_2 h_{2,1} + r_6) + (\beta_{2,4} g_{2,1}^* + z_2) h_{1,2} - (\beta_{3,4} g_{3,4}^* + z_7) h_{4,3} + g_{4,3} (a_4 h_{4,4}^* + r_8) + (\beta_{4,4} g_{4,4}^* + z_8) h_{3,4} - g_{4,3} (a_3 h_{3,3}^* + r_7) (7)
\]

Where \( z_a \) and \( r_x, \forall x \in \{1, 2, 3, 4, 5, 6, 7, 8 \} \), are mutually uncorrelated, zero-mean Gaussian random variables. Derivation of (6) and (7) are found at the Appendix C.

By analyzing Eqs. (6) and (7), it is obvious that interference factor \( E[b] \) and gain factor \( E[a] \), which are related to the final interference coefficient, are dependent on \( \beta \), the autocorrelation of the estimation error, therefore being dependent on the channel estimation method applied.

In Figs. 1 and 2, respectively, the normalized Interference Factor \( b \) and the Gain Factor \( a \) are found as a function of the autocorrelation of the estimation error \( \beta \) and the mean square error \( \gamma \). By analyzing these figures, it can be noticed that, for higher values of \( \beta_m \), the interference factor \( b \) slightly decreases while the gain factor \( a \) has a slight increment, for the same values of channel estimation’s mean.
square error $\Upsilon$. The behavior of these two factors, qualitatively, tells us the advantage of using high-autocorrelated channel estimation methods (leading to higher $\beta_{\text{m,u}}$ values), although it does not give us a clear figure of how important these slight improvements will be to reduce the final interference coefficient. Thus, the previous mathematical analysis requires a completion which could be achieved through computational evaluation. The computation of the final interference is shown in Fig. 3.

As expected, Fig. 3 indicates that for the same values of $\Upsilon$, the higher $\beta$ is, the smaller is the final interference coefficient influencing the final system performance. Therefore, the use of polynomial interpolation methods is favored. In Appendix D the reader can enjoy a mathematical analysis of the relationship between the autocorrelations of the channel estimation and channel estimation error processes.

3. Proposed Estimation Method

In this section we propose a channel estimation method consisting of an adaptive length frame-based higher order Lagrangean estimator providing CSI to the Orthogonal Linear Maximum Likelihood Decoder [1] applied to OFDM technology. Here, it should be noticed that the proposed channel estimation method can be used with any decoder, thus not being restricted to a specific decoding scheme, whatsoever. We describe the system based on a more realistic time-selective, though frequency flat channel of a single OFDM subcarrier.

Since mobile units (MU) are on the move, the severity of the fading they are exposed to is constantly changing. Thus, a channel estimation method which adjusts its frame length according to the Doppler frequency seen by the MUs, avoiding interpolation divergence, will obviously be more desirable than fixed length methods.

The estimation method is subdivided in two parts: The training mode to acquire $h_{\text{m,u}}$ values at the pilots and the adaptive length frame-based interpolation mode. In the scope of this paper we are focusing only on the second one, letting the first mode be provided by any known method such as the ones developed in [13].

Figures 4 and 5 illustrate the structure of the STBC-OFDM system with the proposed channel estimation method and the adaptive frame transmitted on every subcarrier of the OFDM system, respectively. For a $m$ transmit antennas system, the serial input bits are modulated and Space-Time Block Coded into $f_n$ parallel streams. After estimating the Doppler frequency $f_d$ both the transmitter and receiver know the most appropriate frame length to be

![Fig. 1 Normalized Interference factor $b$, after first step of partially orthogonal combiner, for different values of $\beta$ (autocorrelation) and mean square error values, for a four transmit antennas system.](image1.png)

![Fig. 2 Normalized Gain Factor $a$, after first step of partially orthogonal combiner, for different values of $\beta$ (autocorrelation) and mean square error values, for a four transmit antennas system.](image2.png)

![Fig. 3 Interference coefficient for different values of $\beta$ (autocorrelation) and mean square error values, for a four transmit antennas system in noise.](image3.png)

![Fig. 4 Basic schematic diagram of the proposed system.](image4.png)
adapted during the transmission. This is due to the fact that the Doppler frequency is a consequence of the relative motion between the transmitter and receiver, thus the \( fd \) estimated by both parts should be the same. After the whole frame arrives at the receiver, an interpolation is run based on the channel values acquired from the pilots (not STBC encoded) which were placed at the pre-amble and post-amble of the frame during the transmission. These channel estimates are then provided to the Linear Maximum Likelihood Decoder, which yields the symbol estimates.

Figure 5 indicates the position of the pilots, whose number vary according to the order of the interpolation (precision) adopted and the data part, which is composed by symbols \( s_m \) belonging to the same subcarrier of the same transmit antenna. Since in a mobile scenario the fading statistics of the mobiles are constantly changing, a new estimation of \( fd \) (out of the scope of this work) is then necessary to assure that the frame-length to be utilized is the most appropriate for the subsequent transmission.

3.1 Lagrange Interpolation

For a given set of data points \((t_i, h_i)\) where \(t_i\) represents the pilot time-slots (TS) and \(h_i\), the real/imaginary channel values at the same TS, the Lagrange interpolating polynomial for channel estimation is given by:

\[
P(t) = \sum_{i=1}^{n+1} P_i(t)
\]

where

\[
P_i(t) = h_i \prod_{k=1, k \neq i}^{n+1} \frac{t-t_k}{t_i-t_k}
\]

\(P(t)\) is our \(n\)th order polynomial which passes through the points \(h_1, h_2, \ldots, h_{n+1}\). Due to the nature of our application, the pilots are bunched to both extremities of our frame not letting Lagrange Interpolation suffer from Runge's Phenomenon, as seen in [12]. This prevents our channel estimation from diverging wildly from the actual channel as it approaches the data points, even when dealing with higher-order interpolations.

3.2 Frame Size Determination

In a mobile environment, in order to achieve a precise channel estimation based on interpolation, it is necessary to have a certain criterion on how to determine the frame size in which the interpolation will occur. For low fading rates, time-selectivity is not so severe, thus longer frames should be employed. On the other hand, in high fading rate scenarios shorter frames yield higher estimation accuracy, though reduce the system throughput.

Our criterion for assuring no interpolation divergence is to employ frames allowing either one peak or one trough (fade) of the actual real/imaginary channel within its length. However, the challenge is how to determine this length in a simple and effective way, which could be implemented in low cost and small sized terminals. Since the in-phase and quadrature channels are interpolated separately, we can solve this problem by using the level-crossing ratio theory of Gaussian stochastic processes. As described by [15], the expected number of zeros crossings of a stationary Gaussian process with twice differentiable autocorrelation function, \(r(\tau) = E[h_m h_{m+\tau}]\) is given by

\[
E[N] = \frac{T}{\pi} \left( \frac{-r''(0)}{r(0)} \right) \left( \exp \left( \frac{-\mu^2}{2r(0)} \right) \right)
\]

where \(\tau\) represents the instant difference of the same channel, \(N\) is the number of zero (level) crossings, \(T\) represents the considered time length and \(\mu\) is the mean process value. Since peak/trough of the original stochastic Gaussian process (imaginary/real part of the channel) are equivalent to zeros of the derivative of the original process, our task reduces to determining the expected number of zero crossings of the differentiated process \(E[N_d]\), since the theoretical frame length \(FL\) can easily be calculated by

\[
FL = \frac{1}{E[N_d]}
\]

Also, differentiation is a linear process, therefore the derivative of a stochastic Gaussian process remains a stochastic Gaussian process. Although, the autocorrelation of the differentiated process becomes

\[
r_d(\tau) = J_0(2\pi f_d \tau \Delta t)\sigma_d^2
\]

where \(J_0\) is the Bessel function of the first kind, \(\sigma_d^2\) refers to the variance of the process after the differentiation, \(f_d\) is the maximum Doppler frequency and \(\Delta t\) is a constant representing the time lag between consecutive symbol transmissions, in other words, the transmit symbol duration.

Let \(T\) be the unity of time and reminding that \(\mu\) is zero for both the process and its derivative, (10) can be reduced to

\[
E[N_d] = \frac{1}{\pi} \left( \frac{-r''(0)}{r_d(0)} \right)
\]

where \(r_d''(0)\) can be calculated as follows: let

\[
C = 2\pi f_d \Delta t
\]

then
where, due to the time-selectivity, $I(\cdot)$ becomes

$$I(\delta, g(\rho)) = \frac{\pi (1-\delta)}{\pi} \int_0^{\pi (1-\delta)} \left( \mu_{\gamma \rho} \left( \frac{g}{\rho \cdot \sin^2(\theta)} \right) \right)^2 \left( \mu_{\gamma \rho} \left( -\frac{g}{\rho \cdot \sin^2(\theta)} \right) \right)^2 d\theta,$$

with

$$g_{\text{psk}}(\delta) = \sin^2(\pi \delta).$$

and

$$\delta_k^* = \frac{2k-1}{M},$$

$$\delta_k^* = \frac{2k+1}{M}$$

for uniform PSK constellations, in which $M$ represents the adopted constellation size.

Also, $\mu_{\alpha \gamma}(s)$ and $\mu_{\alpha \Gamma}(s)$ are the Moment Generating Function MGF of the SNR per bit $\gamma_n$ associated with the rayleigh fading path $n$ affected by $J_0(2\pi f d 3\Delta t)$ and $J_0(2\pi f d 5\Delta t)$, respectively:

$$\mu_{\alpha \gamma}(s) = \left( 1 - s \frac{\alpha_3}{4} \gamma_n \right)^{-1}$$

$$\mu_{\alpha \Gamma}(s) = \left( 1 - s \frac{\alpha_5}{4} \gamma_n \right)^{-1}$$

Notice that due to time-selective channels, the MGF functions $\mu_{\gamma n}$ provided by [16] need to be slightly modified. Here, the differences lie in the inclusion of the factors $\alpha_3$ and $\alpha_5$ multiplying the $\gamma_n$ (degrading $\gamma_n$ since $\alpha_u < 1$, $\forall u \neq 0$) and the number 4 dividing it ($n_4 = 4$), since the transmit power is normalized at the transmission. Also, $J_0$ represents the Bessel function of the first kind.

The theoretical BER of conventional systems in time-selective fading channels suffer from loss of orthogonality in symbol decoding, due to the presence of an interference factor, say $\zeta_n$, while keeping the maximum diversity gain. Thus, the derivation of the BER for those systems can be easily computed in the same manner as for the BER of the proposed one, except for the fact that

$$I(\delta, g) = \frac{\pi (1-\delta)}{\pi} \int_0^{\pi (1-\delta)} \left( \mu_{\gamma \rho} \left( \frac{g}{\rho \cdot \sin^2(\theta)} \right) \right)^2 \left( \mu_{\gamma \rho} \left( -\frac{g}{\rho \cdot \sin^2(\theta)} \right) \right)^2 d\theta,$$

and its MGF, associated with the non-correlated and balanced paths, is given by

$$\mu_{\gamma n}(s) = \left( 1 - s \frac{1}{4} \gamma_n \right)^{-1}$$

where $\gamma_n$ is the Signal-to-Interference-plus-Noise Ratio (SINR), whose derivation is given in [18]. For a four transmit-one receive antenna scheme of the conventional system with normalized transmit power, $\zeta_n$ can be calculated by

$$\mu_{\gamma n}(s) = \left( 1 - s \frac{1}{4} \gamma_n \right)^{-1}$$
\[ \zeta_n = \frac{1}{8} (1 - \alpha_n^3) + \frac{1}{4} (1 - \alpha_n^3) + \frac{3}{8} (1 - \alpha_n^3) + \frac{3}{8} (1 - \alpha_n^3) + \frac{1}{4} (1 - \alpha_n^3) + \frac{1}{8} (1 - \alpha_n^3). \]  

Here, the \( \alpha_{m,n} \) notation was simplified to \( \alpha_n \) since all channels have the same normalized autocorrelation function, given by \( J_0(2\pi \cdot f \cdot \Delta t) \).

Since the channel model, as in (3), holds for fading rates ranging from low to moderate [7], it is obvious that for very high fading rates the BER formulas will lose their accuracy.

5. Performance Analysis and Simulations

In this section we show some relevant results confirming the efficacy of the proposed technique. We propose a 30 Mbits/s Space-Time Block Coded OFDM system with central frequency of 5 GHz, 1024 subcarriers, each transmitting at a data rate of 29295 bits/sec, 8-QPSK modulation and having the Adaptive Frame-Based Interpolation Method as the channel estimator.

The overall OFDM fading channel being considered in this paper is both time and frequency selective. The autocorrelation of the channel in the time (across successive symbols) and frequency (across adjacent carriers) domains, as shown in [19], can be respectively given by

\[ r_t(\Delta t) = J_0(2\pi \cdot f \cdot \Delta t) \]  

\[ r_f(\Delta f) = J_0(2\pi \cdot f_d \cdot \Delta f), \]

where \( J_0 \) is the Bessel function of the first kind, \( \Delta t \) represents the delay spread of the channel and \( f, f_d \), the maximum Doppler frequency. As in [1], correlation in the space domain (across different antennas) as well as OFDM co-channel interference due to carrier off-set, symbol distortion, cross talks or other effects, are not being considered.

For the proposed system the symbol duration in each subcarrier is approximately 0.1024 ms \( (B_s \approx 9.765 \text{ kHz}) \) while the typical delay spread for urban environments is 3 \( \mu s \) \( (B_c \approx 53 \text{ kHz}) \) [20], where \( B_s \) and \( B_c \) are the signal bandwidth and the channel coherence bandwidth, respectively. This makes each subcarrier of the OFDM system to experience a frequency-flat channel. However, as described by [1], channel continuous (rather than block) fluctuations along the time become significative specially in a context where O-STBC is being used.

In addition to this, O-STBC with four transmit antennas was used as the coding scheme and various Lagrange interpolation orders were adopted for the proposed system. Simulations utilizing Kalman-filter (KF) as the channel estimator where implemented with 100 KF iterations.

In Fig. 6 we can see that, despite its reduced complexity, the use of 7th order Lagrange interpolation providing channel estimation to the proposed system achieves an improved performance when compared to the conventional decoder using Kalman filter and even when the latter benefits from perfect channel knowledge. Moreover, performance improvement is not expressed only by BER reduction, but also by an increase of the system throughput, since KF needed the insertion of pilots every 32 symbols to avoid wild divergence. Also, it can be observed that the 7th order interpolation yields a very accurate channel estimation which does not significantly affect the performance of the proposed system even when compared to the case of perfect channel knowledge. As the interpolation order is reduced (maintaining the frame-size), more severe degradation can be observed. In this figure performance degradation causing an error floor for the case of 3rd and 4th interpolation orders can be observed. Obviously, this is due to the fact that a lower order interpolation can not estimate the channel as precisely as a higher order does. By examining Figs. 7 and 8, the difference between the quality of channel estimation provided by the proposed method and the online channel tracking (Kalman filter), becomes obvious.

Figure 9 shows BER comparisons of the proposed system using the adaptive frame-based interpolation method as well as the conventional system using Kalman filter for different pilot placement configurations. As can be observed, for frame sizes calculated in accordance to (20), even a 5th order interpolation yields very accurate channel estimation,
thus the performance degradation is very small when compared to the case of perfect channel knowledge. Also, it can be observed the effect of increasing the throughput (increasing the number of data symbols between pilots) in a Kalman filter based channel estimation. In addition to this, even for the case of reduced throughput where 1 pilot is inserted every 16 symbols (8 symbols effectively transmitted for a half-rate code), the performance degradation is significant when compared to the case of perfect CSI at the decoder, for high SNR scenarios.

As a complement to the analysis in Sect. 2, we provide Fig. 10, which shows the BER comparison of the proposed system using the adaptive frame-based interpolation and a channel tracking-like method as the channel estimator, both with the same MSE. Here, it should be understood that the tracking-like is a fictitious channel estimation method and that it was created as a way to obtain BER comparisons between high-correlated and low-correlated estimation methods with the same MSE. The tracking-like method was generated in the following manner. The MSE $\varepsilon$ of the smooth channel estimates generated by the proposed adaptive frame-based interpolation method is calculated for every time-slot of a frame. Then, by providing these $\varepsilon$ to Eq. (2), from Sect. 2, tracking-like channel estimates are generated for each time slot $n$, resulting in a low-autocorrelated estimation with exactly the same MSE of the proposed method.

Figure 11 shows a BER comparison for different fading rates. This plot also emphasizes the reliability of the proposed adaptive channel estimation scheme, which bases its frame length determination on Eq. (20). For the whole considered range (static to moderate fading), the conventional system faces considerably performance loss, while the proposed one shows robustness to time-selectivity. Again, no significant degradation of performance can be seen by the use of the adaptive frame-based interpolation method when compared to the case of perfect channel knowledge. More-
Fig. 12 BER comparison of systems with simulated and theoretical BER (8-PSK and $fda = 0.0051$).

Fig. 13 BER comparison of systems with simulated and theoretical BER (8-PSK and $fda = 0.0077$).

over, from Figs. 12 and 13, it can be seen that for moderate fading channels, the provided theoretical BER formulas are very accurate for the proposed system and good approximations for the conventional one. The reason for this difference of accuracy is that $P_b(E)$ for the conventional system is computed based on the SINR, which is dependent on the interference factor $\zeta$, whose $u$ assumes high values for the case of four transmit antennas.

6. Conclusions

An efficient and low complexity adaptive length frame based interpolation method which significantly improves the quality of channel estimation in the constantly changing scenario of mobile communication systems is proposed, allowing MUs to seamlessly range from low to high fading rates situations. As a consequence, the problem emergent from the application of O-STBC as the encoding scheme in the presence of “moderate” fading channels is combated. Moreover, the proposed system clearly outperforms the conventional one adopting the computationally expensive Kalman filter, and even when it benefits from perfect CSI.

The adaptive frame-based interpolation method also makes a more effective use of the pilots transmitted during the training mode. Among these pilots used to acquire the $h_{m,n}$ values, the $\frac{m+1}{2}$ pilots at both frame extremities (refer to Fig. 5) provide the interpolation of the whole frame. On the other hand, for a Kalman filter method, the training mode pilots are used to acquire only the initial $h_{m,n}$ value, on which the decoder will fully rely to make further iterations. This process will repeat every few time-slots periods ahead, before Kalman filter diverges from the actual channel.

Very accurate theoretical formulas for the BER computation of the proposed system and well approximated ones for the conventional system, in the presence of time-selective fading channels, were also derived.

References

Appendix A

\[ g_{m,n} = h_{m,n} - \bar{h}_{m,n} \]  
\[ \Upsilon = E[|h_{m,n} - \bar{h}_{m,n}|^2] = E[|g_{m,n}|^2] \cdot \sigma^2_{\eta_{\nu,n}} \]  
\[ \sigma^2_{\nu_{\eta,n}} = (1 - \epsilon)\sigma^2_{\nu_{\eta,n}} + \sigma^2_{\eta_{\nu,n}} \]

Where \( E[\cdot] \) denotes expected value. Let

\[ \Upsilon = \epsilon \sigma^2_{\nu_{\eta,n}} \]

Then, \( \epsilon \) is the normalized mean square error. We can verify the consistency of (2), by checking its variance (power) equivalence

\[ \sigma^2_{\nu_{\eta,n}} = (1 - \epsilon)\sigma^2_{\nu_{\eta,n}} + \sigma^2_{\eta_{\nu,n}} \]

Appendix B

As described by [7], if the fading rate is not so severe, then (3) and (4) hold. The variance of \( \nu_{m,n} (\sigma^2_{\nu_{m,n}}) \) and \( \eta_{m,n} (\sigma^2_{\eta_{m,n}}) \) are given by \((1 - \sigma^2_{\eta_{m,n}})\sigma^2_{\eta_{m,n}}\) and \((1 - \beta^2_{m,n})\Upsilon\), respectively. Consistency of (3) can be checked by its variance equivalence

\[ \sigma^2_{\nu_{\eta,n}} = \beta^2_{m,n}\sigma^2_{\nu_{\eta,n}} + (1 - \beta^2_{m,n})\sigma^2_{\eta_{m,n}} \]

As expected the channel variance does not depend on the time instant \( n \), thus its notation can be simplified to \( \sigma^2_{\nu_{\eta,n}} \).

Consistency of (4) can be checked by the same procedure

\[ \sigma^2_{\eta_{\nu,n}} = \beta^2_{m,n}\Upsilon + (1 - \beta^2_{m,n})\Upsilon \]

\[ \Upsilon = \sigma^2_{\eta_{\nu,n}} = \sigma^2_{\eta_{\nu,n}} \]

Appendix C

By simply rearranging (1), gain factor \( a \) and interference factor \( b \) can be, respectively, written as

\[ a = h_{1,1}\tilde{h}_{1,6} + h_{2,1}\tilde{h}_{1,7} + h_{3,1}\tilde{h}_{1,8} + h_{4,1}\tilde{h}_{1,9} \]

\[ b = h_{1,2}\tilde{h}_{1,6} - h_{1,6}\tilde{h}_{1,2} - h_{2,3}\tilde{h}_{1,4} + h_{3,1}\tilde{h}_{1,7} \]

For the case of perfect CSI at the receiver, Eq. (A-7) becomes

\[ a = h_{1,1}h_{1,6} + h_{2,1}h_{2,5} + h_{3,1}h_{3,6} + h_{4,1}h_{4,5} \]

Making use of the models described in (3) and (4)

\[ h_{1,1}\tilde{h}_{1,6} = h_{1,1}\sqrt{1 - \varepsilon}h_{1,6} + g_{1,6} \]

\[ h_{1,1}\tilde{h}_{1,7} = h_{1,1}(\beta_{1,1}g_{1,7} + 1) + h_{1,6}\tilde{h}_{1,6}\sqrt{1 - \varepsilon} \]

\[ h_{1,2}\tilde{h}_{1,6} = h_{1,2}(\sqrt{1 - \varepsilon}h_{1,6} + g_{1,2}) \]

\[ h_{1,2}\tilde{h}_{1,7} = h_{1,2}(\sqrt{1 - \varepsilon}h_{1,6} + g_{1,2}) \]

Where where, \( \varepsilon \) and \( r_{c} \), \( \forall \kappa \in \{1, 2, 3, 4, 5, 6, 7, 8\} \), are mutually uncorrelated, zero-mean Gaussian random variables with variance \( \sigma^2_{\nu_{\eta,n}} \) and \( \sigma^2_{\eta_{m,n}} \), respectively. By modeling all elements of (A-7) and (A-8) as done in (A-10) and (A-11), (6) and (7) are derived.

Appendix D

\[ \gamma_{m,n} = \frac{1}{\sigma^2_{\nu_{\eta,n}}} E[h_{m,n}g_{m,n}] \]

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Above, \( \gamma_{m,n} \) is the normalized autocorrelation function of the estimated channel, given by \( J_{0}(2\pi f du) \), where \( J_{0} \) represents the Bessel function of the first kind, \( f \) is the maximum Doppler frequency, \( \Delta t \) is the transmit symbol duration and \( u \), the time instant difference. From (A-12) we can clearly notice the relationship between the autocorrelation function of the channel estimation process \( \gamma_{m,n} \) and of the channel estimation error process, \( \beta_{m,n} \). Therefore we can conclude that for fixed values of \( \alpha_{m,n} \), higher values of \( \gamma_{m,n} \) (as the ones generated by polynomial interpolation methods)
yield higher values of $\beta_{\text{max}}$, generating smaller interference coefficient as indicated by Fig. 3. This will consequently lead to less degradation of the final system performance.

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