LETTER

Cumulative Decision Feedback Technique for Energy Constrained Wireless Sensor Networks

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SUMMARY The application of Cumulative Decision Feedback (DF) technique for energy/complexity constrained Wireless Sensor Networks (WSN) is considered. Theoretical bit error probability and average rate of a BPSK modulated DF are derived together with PHY-MAC layers’ energy efficiency model for DF and Forward Error Correction (FEC) techniques. Moreover, an empirical optimization, which in turn relies upon a low complexity SNR estimation method also derived in this letter, is applied to the DF technique in order to obtain maximum energy efficiency.

key words: sensor networks, decision feedback, FEC, packet combining, energy efficiency

1. Introduction

One of the most crucial topics under research for WSN is energy efficiency since in many applications battery replacement of a sensor node is not desired or even possible. A myriad of energy efficient approaches can be found in the literature for network (e.g. geographic routing protocols), MAC (e.g. schedule-based and contention-based protocols) and physical layer, with the latter dealing with the fundamental question of “coding or not coding?” since WSN are short-range applications where the circuit energy consumption is comparable or even dominates the transmission energy [1].

In [2], various FEC schemes have been analyzed for multi-hop WSN and it was concluded that they can be energy inefficient since the power consumed by the decoders compared with the energy savings due to coding gain is very dependent on the scenarios in which sensors are operating. However, single-hop configurations of WSN [3] indeed relax the aforementioned problem by transferring the complexity of decoding to the sink node which is neither energy nor complexity constrained.

Although divergent regarding the best configuration to be adopted in WSN, the aforementioned approaches (and the references therein) share a limitation in their analysis of what is an energy efficient/inefficient scheme for WSN. All conclusions are drawn based on the values of required energy per bit which yields the desired BER at the receiving side. However, in many WSN applications there should be few or no human planning, such as link budget to calculate the required transmit power which yields the desired BER for the network. On the contrary, nodes are scattered on the environment to be monitored (e.g. volcanoes, forests), forming links with different SNR and are expected to preserve their energy and consequently increase the network lifespan. In light of what has been discussed above, we are motivated to find an adaptive, complexity-inexpensive technique which could be easily embedded to the nodes in either single-hop or multi-hop configurations of WSN.

In this letter we analyze the performance of the DF technique [4], [5] applied to WSN. Rather than focusing on energy efficiency of decoders, which are manufacturer design dependent (e.g. gate counts of decoder’s circuitry, clock frequency, leakage current reduction techniques [3]), we focus on another important cause of energy waste resultant from the need to retransmit packets which are not correctly decoded [6]–[8]. We derive a joint PHY-MAC layers’ energy efficiency model for WSN employing no coding, DF, Simple Packet Combining (SPaC) [8] as well as FEC, such as the well known (8,4) extended Hamming Code and the systematic (16,8) linear block codes [9], [10]. We, then, empirically optimize DF relying on a low complexity SNR estimation for BPSK modulated motes, also derived in this letter. Moreover, we point out the operation region, in terms of node-to-node distance, in which one technique is more energy efficient than the others as well as discuss the reasons for their efficiency/inefficiency.

2. System Configuration

2.1 Cumulative Decision Feedback Systems

Cumulative Decision Feedback (DF) systems [4], [5], hereafter addressed simply by DF systems, to the best of the authors’ knowledge, have never been proposed in the context of WSN. DF systems utilize a null zone detector, shown in Fig. 1, which withholds its decision in doubtful cases, characterized by a signal $y(t)$ falling within the null zone

$$y(t) = s_i(t) + n(t), \, 0 \leq t < \infty, \, i \in \{1, 2\}, \quad (1)$$

where $s_i(t)$ is chosen from a BPSK constellation and $n(t)$ is the additive white Gaussian noise with zero mean and $\sigma_n^2 = N_0/2$ per dimension. For convenience of notation we shall denote $\{y(t), 0 \leq t < \infty\}$ by $y$. The receiver, then,
informs the transmitter about the withholding by a feedback channel and makes a request for retransmission of the doubtful signal. Upon the reception of the retransmitted signal, the decoder averages it with the previous version of the signal. The decoder can repeat this process until the averaged signal achieves a predetermined confidence level, determined by \( \alpha \), thus executing

\[
\hat{s} = \sum_{r=0}^{\infty} \frac{y_{rr}}{r+1}, \quad \{0 \leq r < \infty\},
\]

which can be put on the form of

\[
\hat{s} = s_1 + \frac{n_1 + \cdots + n_{r+1}}{r+1},
\]

before outputting the final symbol estimation \( \hat{s} \). Observe that by executing (3), DF system averages the noise, thus reducing \( \sigma_n^2 \):

\[
\sigma_{n+1}^2 = \sigma_n^2 + \cdots + \sigma_{n+1}^2 = \frac{\sigma_n^2}{(r+1)^2} + \cdots + \frac{\sigma_{n+1}^2}{(r+1)^2} = \frac{\sigma_n^2}{r+1}.
\]

Also, note that one should limit \( r \), the number of repetitions, to a practical maximum value since there is a nonzero probability that the averaged signal will never reach the predetermined confidence level, thus never moving out of the null zone.

One very attractive characteristic of DF is its adaptability to the scenario, extremely important in the context of WSN. Whenever the instantaneous SNR of a given link is low, DF will likely ask for retransmission, thus reducing the probability of erroneous detection. This on demand characteristic of DF allows the motes to use their energy according to their instantaneous needs. In addition to it, the complexity of a DF system is comparable to the one of an uncoded system since it does not make use of encoder/decoder. Also, DF does not impose energy expenditure with overhearing, as happens with SPaC, and does not have the burden of always transmitting parity bits (redundancy), as in FEC approaches.

Given that \( s(t) = \sqrt{E_s} \) was transmitted and that \( t = 0 \), we derive in (5), the bit-error probability \( P_e \) from node-to-node of a network employing DF system.

\[ P_e = \int_{-\infty}^{\infty} p(y_0|s)dy_0 + \int_{-\infty}^{\alpha} \int_{-\infty}^{\alpha} p(y_0|s) \cdot p\left(\frac{y_0 + y_1}{2} | s\right) dy_0 dy_1 + \cdots + \int_{-\infty}^{\alpha} \cdots \int_{-\infty}^{\alpha} p(y_0|s) \cdot p\left(\frac{y_0 + y_1 + \cdots + y_r}{r+1} | s\right) dy_0 dy_1 \cdots dy_r \]

(5)

2.2 Network Configuration

Although WSN will be composed of many nodes, we focus on a very small linear segment of the network composed by three nodes, as in Fig. 2, in order to compare the energy efficiency of the analyzed techniques. By identifying the least energy expensive configuration for a given node-to-node distance, we find the method which yields the lowest transmit energy per correctly decoded bit. Since the whole network can be viewed as a group of many linear network segments with different distance among nodes, by improving the energy efficiency of each segment the overall networks’ efficiency will be raised.

3. PHY-MAC Layers’ Energy Efficiency Model

In this section we present a joint PHY-MAC layer energy efficiency model for the considered techniques. Figure 3 shows a simplified version of the packet provided in [11], where the PHY (preamble sequence, start of frame delimiter and frame length) and MAC (frame control, sequence number and addressing fields) header bits are all represented by \( y \). The data payload field is subdivided into \( l \) information bits and \( r \) parity bits.

3.1 Preliminaries

Energy efficiency can be defined as \( \eta = \frac{E_{\text{useful}}}{E_{\text{total}}} \). That is to say, the ratio between the energy spent with the correctly received information bits and the total energy spent with the communication. Differently from \( E_{\text{total}} \), \( E_{\text{useful}} \) is the same for all techniques and, mathematically, defined as

\[ E_{\text{useful}} = (PCKT - PCKT_{\text{error}})E_s, \]

(6)
where \( PCKT \) is the number of transmitted packets and \( PCKT_{\text{err}} \), the number of packets received with error. \( E_b \) is the energy spent to radio-communicate one bit

\[
E_b = E_{tx} + E_{po} + E_{rx},
\]

where the energy spent by the amplifier, \( E_{po} = \frac{P_o}{T} \), is 10 nJ for an output power \( P_o = -20 \) dBm, and according to benchmarks provided by [12], the energies \( E_{tx} = \frac{P_t}{T} \) and \( E_{rx} = \frac{P_r}{T} \) spent by the receiver and transmitter circuitry are, respectively, 1.05 \( \mu \)J and 0.55 \( \mu \)J for a data rate \( R = 19.2 \) kbps.

### 3.2 Modeling Energy Efficiency

Based on SPaC, uncoded, FEC and DF system operation described in the appendix section and the network segments shown in Fig. 2, the \( E_{\text{total}} \) and \( \eta \) of each system can be easily modeled. For the SPaC system,

\[
E_{\text{total}} = PCKT[(\gamma + l)(E_b + E_{rx}) + (\gamma + \tau)E_b + E_{dp}],
\]

where \( P_{\text{ER}} = 1 - (1 - P_e)^l \) with \( P_e \) being the bit-error probability of the system. Repeating the same modeling method yields for the uncoded system

\[
E_{\text{total}} = 2PCKT(l + \gamma)E_b,
\]

\[
\eta_{\text{spac}} = (1 - P_{\text{ER}}) \frac{lE_b}{(\gamma + l)(E_b + E_{rx}) + (\gamma + \tau)E_b + E_{dp}},
\]

while for the FEC system,

\[
E_{\text{total}} = 2PCKT[(\gamma + \tau)E_b + E_{dp}],
\]

\[
\eta_{\text{fecz}} = (1 - P_{\text{ER}}) \frac{lE_b}{2(\gamma + \tau)E_b + E_{dp}}.
\]

For the proposed DF system,

\[
E_{\text{total}} = 2PCKT(\gamma + l + \tau_1)E_b + 2PCKT_{FB}(\beta + \tau_2)E_b,
\]

where \( \tau_1 \) accounts for the retransmitted bits, \( PCKT_{FB} \) represents the total number of feedbacked packets with \( \beta \) header bits, and \( \tau_2 \), the data payload field bits of the feedbacked packet, which informs the transmitter which bits should be retransmitted. Following,

\[
\eta_{\text{df}} = \frac{(1 - P_{\text{ER}})}{2(\gamma + \beta + \tau_1 + \tau_2)}.
\]

### 4. Decision Feedback Parameters Optimization

#### 4.1 Signal to Noise Ratio Estimation

In this subsection we develop a simple SNR estimator for BPSK modulation in AWGN channel based on the running mean/variance of the received signals. Since WSN are energy/complexity constrained, the estimation derived here is not meant to be optimal, however, its loss of performance being offset by reduced computational complexity.

Estimators are mainly divided in two categories, “in-service” and “data-aided” [13]. The former relies solely on the information-bearing bits to perform the estimation. The latter, on training sequences. The SNR estimation developed in this section is of “in-service” nature, thus not imposing upon throughput, and utilizes the \( \gamma \) bits from the header of the PHY and MAC layers to, recursively, execute

\[
\hat{\mu}_y = \frac{1}{T} \sum_{t=0}^{T-1} y_t,
\]

\[
\hat{\sigma}_y^2 = \frac{1}{T} \sum_{t=0}^{T-1} (y_t - \hat{\mu}_y)^2,
\]

\[
\hat{E}_r = \left( \frac{1}{T} \sum_{t=0}^{T-1} |y_t|^2 \right)^2.
\]

where \( y_t \) is given by (1), \( \hat{\mu}_y \) is the estimated mean value of the received signal \( y_t \) and \( \hat{\sigma}_y^2 \), its estimated variance. \( \hat{E}_r \) is a coarse estimation of the received symbol power \( \sqrt{E_r} = \alpha \sqrt{E_s} \). Here, \( \alpha \) is a given pathloss attenuation factor resultant from the distance between sensor nodes, unimportant to the calculations.

The estimated noise variance is, then, given by

\[
\hat{\sigma}_n^2 = |\hat{\sigma}_y^2 - \sqrt{\hat{E}_r}|,
\]

and the estimated SNR can be promptly calculated,

\[
\hat{\rho} = 10 \log \frac{\hat{E}_r}{\hat{\sigma}_n^2}.
\]

It should be observed that the above derived method can only be adopted for BPSK modulated systems because \( s_t \in \{-\sqrt{E_s}, +\sqrt{E_s}\} \). Figure 4 shows the MSE of \( \hat{\rho} \). We can notice that for \( \gamma \approx 400 \) bits (by the fourth received frame), the method is already very precise. In addition to it, it should be stressed that in this letter nodes are considered static, thus \( \hat{\rho} \) should be calculated only during the initialization of the WSN.
4.2 Empirical Optimization

In this subsection the parameters of the DF technique which yield maximum energy efficiency are optimized by means of empirical optimization method. That is to say, the energy expenditure per correctly received bit is minimized. In order to start the optimization, (15) is put in a simpler form

\[
\eta_{df} = \frac{(1 - PER)}{2(l + \tau_1 + \tau_2)} \cdot \frac{1}{\hat{R}},
\]

(21)
since \(\gamma\) and \(\beta\) are header bits, thus not taking part in the optimization process. Also, \(\frac{1}{\hat{R}}\) is the average rate \(\hat{R}\) which can be calculated by \(\frac{1}{Z}\), where \(Z\) is given by (22), given that \(s_i(t) = \sqrt{E_r}\) was transmitted. Since (5) and (22) do not have simple closed form expressions, they are numerically evaluated for a chosen set of \(|\alpha|\) values, \(|\alpha| \in [0, \sqrt{E_r}]\) and accurately approximated by low order polynomials. These polynomials are, then, embedded to the motes allowing for a reduced computational complexity optimization method. The optimization can be summarized as the following simple algorithm:

**Algorithm: Maximizing Energy Efficiency**

1. Estimate the SNR upon the WSN initialization by the method provided in section 4.1.
2. Using the embedded polynomials, calculate \(P_r\) and \(\hat{R}\) for the estimated SNR value.
3. Calculate (21) for all results obtained from step 2.
4. Select the \(\alpha\) which maximizes (21).

Observe that increasing the number of \(|\alpha|\) values will increase the efficiency of the algorithm. On the other hand, a reduced number of \(|\alpha|\) will reduce the number of calculations, thus further reducing the algorithm complexity.

5. Analysis and Simulations

We have adopted through our simulations the log-distance path model, in the 800 – 1000 MHz band, described by [14]

\[
L(d) = L_0 + 10n \log_{10}(d) + X_n,
\]

(23)
where \(L_0\) is the path loss at 1 meter from the transmitter, \(n\) is the decay factor, \(d\) is the distance expressed in meters, and \(X_n\) is a zero-mean log-normally distributed random variable, with standard deviation \(\sigma\), in dB, accounting for shadowing effects. The variables adopted were pertinent to a tall grassy field and its values are listed in [14],[15]. We have set the output power to the very low level of \(-20\) dBm, considered BPSK modulation and noise figure of 13 dB, characteristic of commercially available motes (e.g. MICA2s) in our simulations. For the DF system, the maximum number of transmissions was set to 10 (9 retransmissions). The feedback channel is the instantaneous and noiseless channel model widely adopted in the literature [4],[5] (and the references therein).

In Fig. 5, we vary the distances between motes (thus varying the SNR) as to obtain the normalized energy efficiency of each system based on (11)–(15). We observe that for frames of both 256 and 512 payload information bits in length, the results are very similar. Since frames much longer than the ones considered in this letter are not expected in WSN, the results shown are believed to cover the whole realistic WSN scenario. This figure can be better understood if analyzed in shorter mote-to-mote distance ranges. First, for distances up to 15 [m], all systems (except the rate \(\frac{1}{2}\) FER systems) yield maximum energy efficiency due to a very low BER, which in turns yields no frame loss for the given lengths. For such distances, DF becomes the uncoded system by choosing \(\alpha = 0\) and, consequently, has unitary rate. Based on that, we grasp that for high SNR scenarios the rate of a given system bears a greater importance than its robustness to error. Second, as distance ranges from 15 [m] to 35 [m], DF system outperforms all systems regardless if Hamming (8,4) or two error correction (16,8) was the adopted encoder. In such scenarios, a balance between rate and robustness to error becomes essential to provide higher energy efficiency. Although FEC systems ensure lower frame loss, that is to say, less need of energy expensive frame retransmissions, it comes with the price of always transmitting redundancy bits (rate 1/2). As for the uncoded system, the lack of redundant transmission results in higher
The SPaC scheme is shown to be ing lower PER outgrows the one of preserving higher rate. grasp that for low SNR scenarios, the importance of ensuring lower PER outgrows the one of preserving higher rate. The SPaC scheme is shown to be inefficient over all the three regions analyzed above.

6. Conclusions

We have proposed the incorporation of an empirically optimized DF to WSN due to its reduced complexity and inherent adaptive nature. Simulation results show that the proposed DF improves energy efficiency compared to conventional techniques within practical mote-to-mote distances. Other contributions of this letter are the derivation of theoretical BER and average rate of BPSK modulated DF, joint PHY-MAC layer energy efficiency models for the techniques considered as well as a low complexity SNR estimation technique for energy/complexity constrained BPSK modulated motes.

References


Appendix A: SPaC System Operation

The SPaC technique [8] makes use of FEC properties, however, it does not spend energy transmitting redundant bits through all links. The way it works can be summarized as follows:

- The two-hops away node C overhears the transmission of l information bits from A to its preceding node B, through a bad link (big path-loss attenuation). Rather than discarding the received packet, it buffers the packet in its memory.
- Node B uses a systematic and invertible linear block encoder to construct codewords, based on the detected l information bits, and send a new packet containing their parity bits to node C.
- Upon the reception of the r parity bits, node C constructs and decodes the codewords formed by the concatenation of the l information bits (packet from A) with the r parity ones (packet from B).

The attractiveness of SPaC scheme comes from the fact that error correction can be achieved while not imposing the burden of always transmitting plain information bits followed by their parities, characteristic of FEC, from node-
to-node of the network. However, its efficiency relies on the correcting capabilities of the chosen linear block code as well as the link quality, which is influenced by the distance between nodes, and that significantly affects the system performance. Moreover, the impact of overhearing on the network’s power consumption should not be ignored, since the power consumed by the receiver ranges from 1 to 3 times the one by the transmitter [16].

Appendix B: Uncoded System Operation

Uncoded system operation can be easily summarized as:
- Node A transmits a packet, with \( l \) information bits in the payload, to node B.
- Node B detects and forwards the packet to node C.
- Upon the reception of the packet from B, node C extracts the \( l \) information bits.

Appendix C: FEC System Operation

FEC system operation can also be, easily, described as:
- Node A transmits a packet, with \( l \) information bits and \( r \) parity bits in the payload, to node B.
- Node B decodes and forwards the packet to node C.
- Upon the reception of the packet from B, node C decodes it and extract the \( l \) information bits.

Appendix D: DF System Operation

The DF system being analyzed in this manuscript differs from the conventional feedback systems on how it handles packets exchange. In a conventional feedback system, feedback packet is transmitted upon the reception of a whole packets exchange. In a conventional feedback system, feedbacks from the conventional feedback systems on how it handles the correcting capabilities of the chosen linear block code as well as the link quality remains \( l \) bits since both nodes A and B know when they have, respectively, transmitted and received \( l \) information bits.

Note that the packet length can vary, however, the information length remains \( l \) bits since both nodes A and B know when they have, respectively, transmitted and received \( l \) information bits.

Appendix E: DF Average Rate \( \bar{R} \) Derivation

Let us assume that \( i \in \{-\sqrt{E_b}, \sqrt{E_b}\} \) and that \( s_i \) is equally like. If \( s_i(t) \) is transmitted, \( y(t) = s_i(t) + n(t) \), where \( n(t) \sim \mathcal{N}(0, \sigma^2) \), is the received signal affected by noise. Thus, if \( s_i = +\sqrt{E_b} \), the probability of having a single transmission for the DF system is given by

\[
P_{e_1} = \int_{-\infty}^{-\alpha} p(y_0|x_i)dy_0 + \int_{+\alpha}^{+\infty} p(y_0|x_i)dy_0, \quad (A\cdot1)
\]

since both terms of (A·1) account for the cases when DF system does not ask for a retransmission. The first term, accounts for the erroneous detections while the second term, for the error-free detections of \( s_i(t) \).

The probability of the DF system having two transmissions is given by

\[
P_{e_2} = \int_{-\alpha}^{+\alpha} p(y_0|x_i)dy_0 \left( \int_{-\infty}^{-\alpha} p \left( \frac{y_0 + y_1}{2} \right) dy_1 \right) \]

\[
+ \int_{-\alpha}^{+\alpha} p \left( \frac{y_0 + y_1}{2} \right) dy_1
\]

\[
(A\cdot2)
\]

where (A·2) can be interpreted as the sum of the probabilities that DF system detects with error and error-free, given that the first received signal fell inside the null zone. The probability of having more transmissions is straight forward, however, their calculations are cumbersome.

Following, the total number of transmitted symbols from the source is, obviously, given by

\[
\text{Symb} = P_{e_1} + 2P_{e_2} + 3P_{e_3} + \cdots.
\]

(A·3)

however, the feedbacked symbols from the receiver to the transmitter cannot be forgotten. These symbols are

\[
\text{Symb}_{FB} = P_{e_1} + 2P_{e_2} + \cdots,
\]

(A·4)

yielding a total number of symbols of

\[
\text{Symb}_{total} = P_{e_1} + 3P_{e_2} + 5P_{e_3} + \cdots.
\]

(A·5)

By having \( \text{Symb}_{total} \), we can calculate the average rate \( \bar{R} \) of DF system by \( \frac{\text{Symb}_{total}}{\text{Symb}_{total}} = Z \), where Z is

\[
Z = \int_{-\infty}^{-\alpha} p(y_0|x_i)dy_0 + \int_{-\alpha}^{+\alpha} p(y_0|x_i)dy_0 + \cdots
\]

\[
+ (2r+1) \int_{-\alpha}^{+\alpha} \cdots \int_{-\alpha}^{+\alpha} p(y_0|x_i) p \left( \frac{y_0 + y_1}{2} \right) dy_1 \cdots dy_{r-1}
\]

\[
\cdots p \left( \frac{y_0 + \cdots + y_{r-1}}{r} \right) dy_0 dy_1 \cdots dy_{r-1}
\]
\[
\left(\int_{-\infty}^{-\alpha} p\left(y_0 + \ldots + y_r \mid s_i\right) dy_r + \int_{+\alpha}^{+\infty} p\left(y_0 + y_1 + \ldots + y_r \mid s_i\right) dy_r\right)
\]

(A·6)